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Review Article

Bifurcation Analysis and Multiobjective Nonlinear Model Predictive Control of Drug Addiction Models

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Abstract

Bifurcation analysis and nonlinear model predictive control were performed on drug addiction models. Rigorous proof showing the existence of bifurcation (branch) points is presented along with computational validation. It is also demonstrated (both numerically and analytically) that the presence of the branch points was instrumental in obtaining the Utopia solution when the multiobjective nonlinear model prediction calculations were performed. Bifurcation analysis was performed using the MATLAB software MATCONT while the multi-objective nonlinear model predictive control was performed by using the optimization language PYOMO.

Introduction

Mental health has become a significant focus for researchers and medical doctors in the last decade. Ironically, drug addiction is both cause and effect for the existence of mental health problems. People with mental health issues resort to drugs and drugs in turn lead to mental health problems. Additionally, drug addiction has led to a considerable amount of poverty and crime. It is therefore important to develop strategies to curb drug addiction. The problem of drug addiction has led to computational research to develop reliable techniques to be able to control drug addiction. This work aims to perform bifurcation analysis in conjunction with multiobjective nonlinear model predictive control (MNLMPC) calculations on models involving drug addiction. This paper is organized as follows. First, the background section with the literature review is presented. The bifurcation analysis techniques and the multiobjective nonlinear model predictive control strategies are presented followed by a description of how the presence of singular points affects the MNLMPC calculations. Two drug addiction example problems where MNLMPC calculations are performed in conjunction with bifurcation analysis are presented. It is numerically demonstrated that the presence of bifurcation points in the drug addiction models enables the MNLMPC calculations to converge to the Utopia solution. It is essential to develop rigorous strategies to be able to effectively combat the drug addiction problem minimizing the number of drug addicts and avoiding the wastage of resources. This paper is a step in that direction.

Background

Bae [1] studied the dynamics of tobacco addiction models. Mushayabasa, and co-workers [2-4] performed dynamic and optimal control studies of drug addiction models. Hasan, et al. [5] investigated the effect of having drug rehabilitation centers to combat drug addiction. Islam, et al. [6,7] developed a mathematical analysis of some dynamic Models of drug addiction, while Lavi, et al. [8] studied the dynamics of drug resistance. Nyabadza, et al. [9] and White, et al. [10] modeled the dynamics of crystal meth abuse and heroin epidemics. Rwat and co-workers [11] examined the effect of recycling the recovered individuals back into the population while Donoghoe [12] studied the effect of drugs on global health. Murray, et al. (2007) [13] studied the effect of cannabis on mental health Pluddemann, [14] investigated the use of strategies to, monitor

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alcohol and substance abuse. Recent work of McGinty, et al. [15], Hooker, et al. [16], Butler, et al. [17], Scott, et al. [18] Chang, et al. [19] and Paquette, et al. [20] have demonstrated the devastating effects of drug addiction and the urgent need to combat this problem.

Akanni, et al. [21] Abidemi, et al. [22] and Olaniyi, et al. [23] studied dynamic models involving illicit drug use. All the optimal control work done so far involves single objective minimization. In this work, multiobjective nonlinear model predictive control calculations are performed on drug addiction models in conjunction with bifurcation analysis. It is numerically demonstrated for two problems involving drug addiction that the presence of bifurcation points enables the MNLMPC calculations to converge to the Utopia solution.

The bifurcation analysis and the MNLPMC methods will now be presented followed by an explanation as to why the presence of bifurcation points leads to the MNLMPC calculations converging to the Utopia solution.

Bifurcation analysis

The existence of multiple steady-states (caused by limit and branch point singularities) and oscillatory behavior caused by Hopf bifurcation points) in chemical processes has led to a lot of computational work to explain the causes of these nonlinear phenomena. N MATCONT, [24,25] is a commonly used software to find limit points, branch points, and Hopf bifurcation points. Consider an ODE system.

$$\dot{x} = f(x,\beta) \tag{1}$$

$x \in R^n$ The tangent plane at any point x is $[v_1, v_2, v_3, v_4, ... v_{n+1}]$. Define matrix A given by

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} & \cdots\cdots & \frac{\partial f_1}{\partial x_n} & \frac{\partial f_1}{\partial \beta} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} & \cdots\cdots & \frac{\partial f_2}{\partial x_n} & \frac{\partial f_2}{\partial \beta} \\ \cdots & & & & & & \\ \cdots & & & & & & \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \frac{\partial f_n}{\partial x_3} & \frac{\partial f_n}{\partial x_4} & \cdots\cdots & \frac{\partial f_n}{\partial x_n} & \frac{\partial f_n}{\partial \beta} \end{bmatrix} \tag{2}$$

With β the bifurcation parameter. The matrix A can be written in a compact form as

$$A = [B \,|\, \frac{\partial f}{\partial \beta}] \tag{3}$$

The tangent surface must be satisfied.

$$Av = 0 \tag{4}$$

For both limit and branch points the matrix B must be singular. For a limit point (LP) the n+1th component of the tangent vector v_{n+1} = 0 and for a Branch Point (BP) the matrix $\begin{bmatrix} A \\ v^T \end{bmatrix}$ must be singular., The function $\det(2 f_x(x,\beta) \odot I_n)$ should be zero for a Hopf bifurcation point. $\odot$ Indicates the alternate product while I_n is the n-square identity matrix. A detailed derivation can be found in Kuznetsov [26,27] and Govaerts [28]. Sridhar [29] used Matcont to perform bifurcation analysis on chemical engineering problems.

MNLMPC (Multiobjective Nonlinear Model Predictive Control) method

The Multiobjective Nonlinear Model Predictive Control (MNLMPC) method was first proposed by Flores Tlacuahuaz, et al. [30] and used by Sridhar [31]. This method is rigorous and it does not involve the use of weighting functions nor does it impose additional parameters or additional constraints on the problem unlike the weighted function or the epsilon correction method [32]. For a problem that is posed as

$$\min J(x,u) = (x_1, x_2 \ldots x_k)$$

$$\text{subject to } \frac{dx}{dt} = F(x,u); h(x,u) \le 0; x^L \le x \le x^U; u^L \le u \le u^U \tag{5}$$

The MNLMPC method first solves dynamic optimization problems independently minimizing/maximizing each x_i individually. The minimization/maximization of x_i will lead to the values x_i^*. Then the optimization problem that will be solved is

$$\min \sqrt{\{x_i - x_i^*\}^2}$$

$$\text{subject to } \frac{dx}{dt} = F(x,u); h(x,u) \le 0; x^L \le x \le x^U; u^L \le u \le u^U \tag{6}$$

This will provide the control values for various times. The first obtained control value is implemented and the remaining is discarded. This procedure is repeated until the implemented and the first obtained control values are the same.

The optimization package in Python, Pyomo [33] where the differential equations are automatically converted to a Nonlinear Program (NLP) using the orthogonal collocation method [34] is commonly used for these calculations. State-of-the-art solvers like IPOPT [35] and BARON [36] are normally used in conjunction with PYOMO.

Effect of singularities (Limit Point (LP) and Branch Point (BP)) on MNLMPC

Let the minimization be of the variables P_1, P_2 l result in the values M_1 and M_2. The multiobjective objective function to be minimized will result in the problem.

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$$\min \int_0^{t_f} ((p_1(t)dt) - M_1)^2 + \int_0^{t_f} ((p_2(t)dt) - M_2)^2 = \int_0^{t_f} P(x,t)dt$$

$$\text{subject to } \frac{dx_i}{dt} = g_i(x,u) \quad (7)$$

The Euler Lagrange equation also known as costate equations will be

$$\frac{d\lambda_i}{dt} = -(\frac{\partial P}{\partial x_i} + \lambda_i g_i) \quad (8)$$

λ_i is the Lagrangian multiplier. Taking the derivative of the objective function we get

$$\frac{d}{dx_i}((p_1 - M_1)^2 + (p_2 - M_2)^2) = 2(p_1 - M_1)\frac{d}{dx_i}(p_1 - M_1) + 2(p_2 - M_2)\frac{d}{dx_i}(p_2 - M_2) \quad (9)$$

At the Utopia point both $(P_1 - M_1)$ and $(P_2 - M_2)$ are zero. Hence

$$\frac{d}{dx_i}((p_1 - M_1)^2 + (p_2 - M_2)^2) = 0 \quad (10)$$

The co-state equation in optimal control is

$$\frac{d}{dt}(\lambda_i) = -\frac{d}{dx_i}((p_1 - M_1)^2 + (p_2 - M_2)^2) - g_x \lambda_i$$

$$\lambda_i(t_f) = 0 \quad (11)$$

λ_i is the Lagrangian multiplier. The first term in this equation is 0 and hence.

$$\frac{d}{dt}(\lambda_i) = -g_x \lambda_i$$

$$\lambda_i(t_f) = 0 \quad (12)$$

If the set of ODE $\frac{dx}{dt} = g(x,u)$ has a limit or a branch point, g_x is singular.

This implies that there are two different vectors-values for $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) > 0$ and $\frac{d}{dt}(\lambda_i) < 0$. In between there is a vector $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) = 0$. This coupled with the boundary condition $\lambda_i(t_f) = 0$ will lead to $[\lambda_i] = 0$ which will make the problem an unconstrained optimization problem. The only solution for the unconstrained problem is the Utopia solution.

Results and discussion

In this section, the results of bifurcation analysis and MNLMPC calculations for two problems involving drug addiction are presented. The models used are described in Islam, et al. [7] and Mushayabasa, et al. [4]. The equations for each problem are presented followed by the bifurcation analysis and MNLMPC results.

Problem 1 Islam, et al. [7]

Equations representing problem 1

- $S_a(t)$ represents individuals who are not drug users, but are at a high risk of taking drugs
- L(t)) represents light drug users
- H(t) represents heavy drug users
- $R_v(t)$ represents drug users under treatment in rehabilitation
- Q(t) represents individuals who will never take drugs

The equations are

$$\frac{dS_a}{dt} = r - \alpha S_a H - \mu S_a - u_1 S_a$$

$$\frac{dL}{dt} = \alpha S_a H - \mu L - \beta L - \delta L - u_2 L$$

$$\frac{dH}{dt} = \beta L - \mu H - \gamma H + p_a R - u_3 H$$

$$\frac{dRv}{dt} = \gamma H - \mu R - \theta R - p_a R \quad (13)$$

$$\frac{dQ}{dt} = \theta R - \mu Q + u_1 S_a + u_2 L + u_3 H$$

The model parameters are

$r = 4.25$; $\mu = 0.00561$; $\alpha = 0.002$; $\beta = 0.6$; $\delta = 0.025$; $\gamma = 1.5$; $pa = 0.02$

u_1, u_2, u_3 are the control variables

where

- r represents the recruitment rate of the population
- μ is the natural mortality rate
- α is the interaction rate among the susceptible and light drug users
- β is the effective rate at which light users convert into heavy drug users
- δ the removal rate from addiction without treatment
- γ is the rate at which heavy addicts are being sent to rehabilitation for treatment
- u_1 is the awareness and educational programs
- u_2 is the family-based care
- u_3 represents the effectiveness of rehabilitation centers

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Bifurcation analysis for problem 1

When bifurcation analysis with μ the bifurcation parameter was performed on the equations representing problem 1, a branch point was found a $[S_a, L, H, R_v, Q, \mu]$ values of (782.26, 0.0, 0, 0,0, 0.005433). Figure 1a shows the bifurcation diagram with this branch point.

MLNMPC for problem 1

The MNLMPC of problem 1, $\sum Q(t)$ was maximized and resulted in a value of 2000; while $\sum H(t)$ was minimized and resulted in a value of 0. The multiobjective optimal control problem involved the minimization of $(\sum Q(t) - 2000)^2 + (\sum H(t) - 0)^2$ the subject to the dynamic equation set representing this problem. This resulted in the Utopia point of 0 and the MNLMPC values of the control variables obtained were $[u_1, u_2, u_3]$ = [0.0004, 0.0405, 0.5362]. The MNLMPC profiles are shown in Figures 1a-1i.

Problem 2 Mushayabasa, et al. [4]

Equations representing problem 2

In this problem, the time-dependent variables are

- $S_v(t)$ susceptible individuals
- $I(t)$ light or occasional drug users

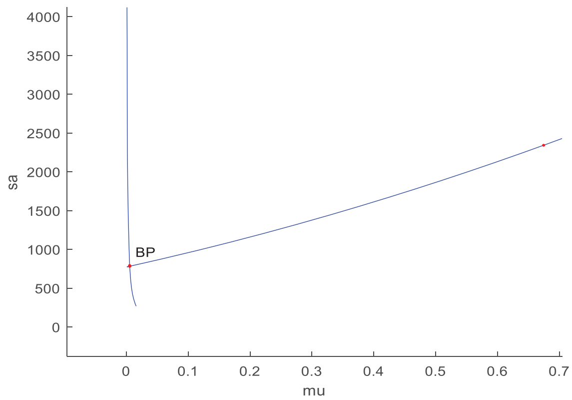

Figure 1a: Bifurcation diagram for problem 1.

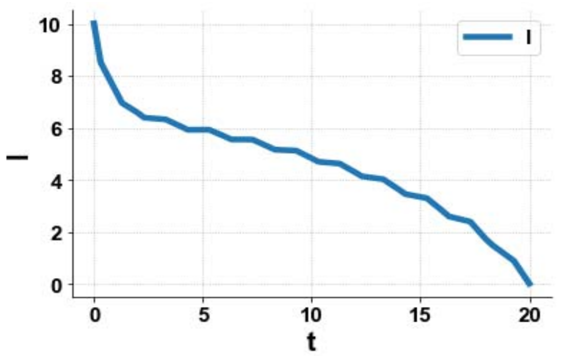

Figure 1b: MNLMPC diagram for problem 1 (I *vs.* t).

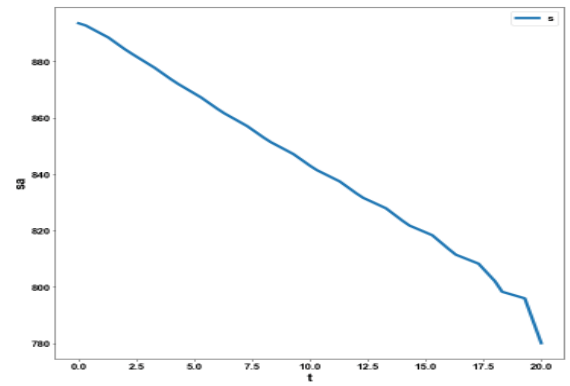

Figure 1c: MNLMPC diagram for problem 1 (sa *vs.* t).

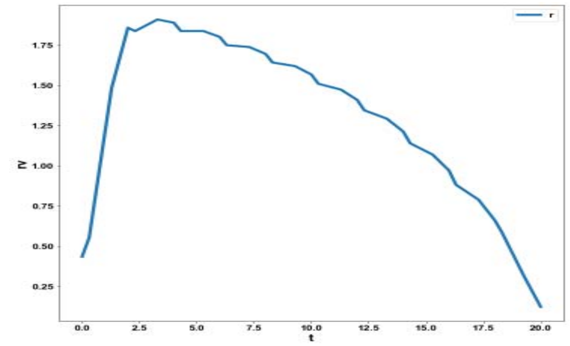

Figure 1d: MNLMPC diagram for problem 1 (rv *vs.* t).

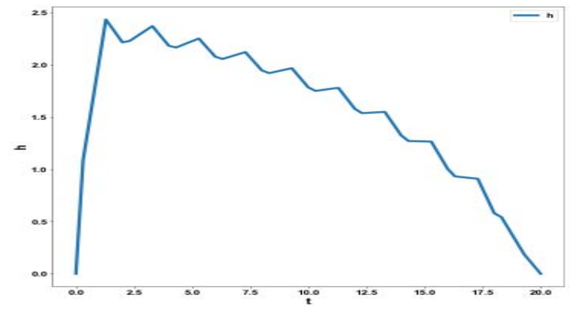

Figure 1e: MNLMPC diagram for problem 1 (h *vs.* t).

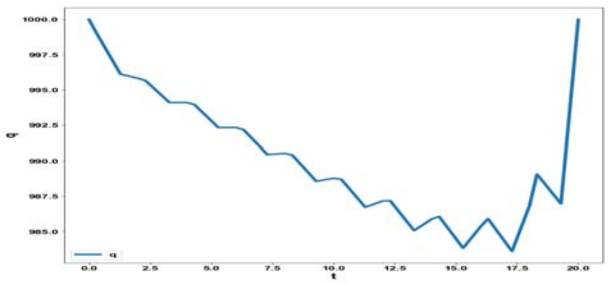

Figure 1f: MNLMPC diagram for problem 1 (q *vs.* t).

- $I_{av}(t)$ heavy drug users
- $M_v(t)$ mentally ill population and (individuals who suffer mental illness due to drug use,
- $R_v(t)$ detected illicit drug users

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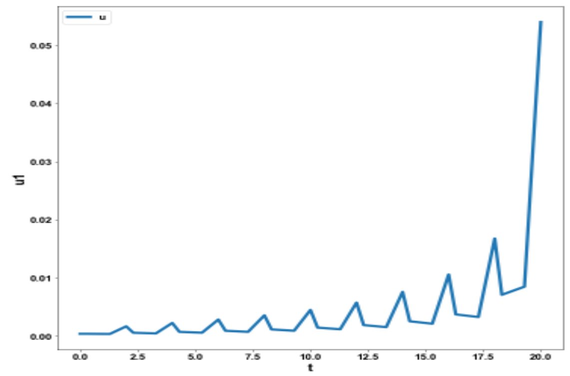

Figure 1g: MNLMPC diagram for problem 1 (u1 *vs*. t).

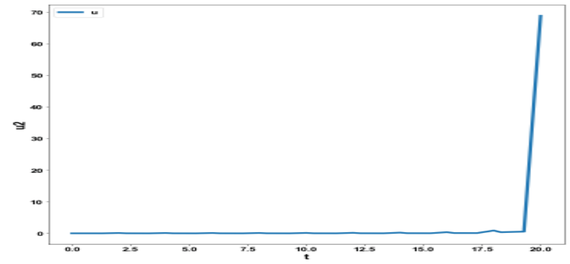

Figure 1h: MNLMPC diagram for problem 1 (u2 *vs*. t).

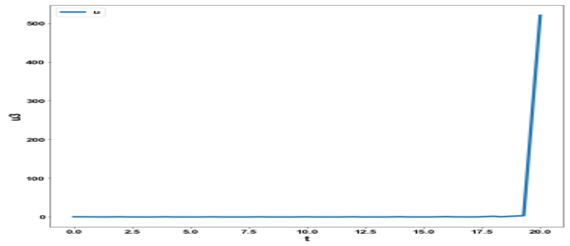

Figure 1i: MNLMPC diagram for problem 1 (u3 *vs*. t).

The equations that represent the drug addiction problem are

$$\frac{dS_v}{dt} = \mu - (1-u_c)\lambda S_v - \mu S_v$$

$$\frac{dI_v}{dt} = (1-u_c)\lambda S_v - (\alpha + \gamma v_c + \sigma + \mu + \psi) I_v$$

$$\frac{dI_{av}}{dt} = \alpha I_v - (\rho v_c + \phi + \mu + d) I_{av}$$

$$\frac{dM_v}{dt} = \sigma I_v + \phi I_{av} - (\mu + \varepsilon v_c + \delta) M_v \tag{14}$$

$$\frac{dR_v}{dt} = v_c(\gamma I + \rho I_{av} + \varepsilon M_v) - (\mu + \omega) R_v$$

$$\lambda = \beta(I_v + k I_{av})$$

And the parameter values are,

$\omega = 0.3$; $\mu = 0.02$; $k = 1.25$; $\beta = 0.35$; $\gamma = 0.1$; $\rho = 0.35$; $\varepsilon = 0.6$; $\alpha = 0.01$; $\psi = 0.03$; $\delta = 0.14$; $d = 0.2$; $\sigma = 0.05$; $\varphi = 0.09$;

u_c, v_c are the control variables

Here,

- α represents the rate at which light drug users become heavy drug users
- γ, ε, ρ the rates of detection and rehabilitation of individuals in classes I_v, M_v, I_{av}
- σ, ϕ the rates at which light and heavy illicit drug users develop mental illness
- ψ, d the permanent exit rates of light and heavy users
- δ mentally ill individuals who permanently exit the model because of death
- ω the rate at which individuals recover as a result of rehabilitation
- β the strength of interactions between susceptible individuals and illicit drug users
- u_c represents the reduction of the intensity of "social influence"
- v_c models the effort on the detection of illicit drug users

Bifurcation analysis for problem 2

When bifurcation analysis with α as the bifurcation parameter was performed on the equations representing problem 2, a branch point was found at $[S_v, I_v, I_{av}, M_v, R_v, \alpha]$ = [1.0, 0.0, 0.0, 0.0, 0.0, 0.430112]. The bifurcation diagram is shown in Figure 2a.

MLNMPC for problem 2

For the MNLMPC of problem 2, $\sum I_v(t)$ and $\sum I_{av}(t)$ were minimized individually and both the minimizations resulted in a value of 0. The multiobjective optimal control problem involved the minimization of $(\sum I_v(t))^2 + (\sum I_{av}(t))^2$ the subject to the dynamic equation set representing this problem.

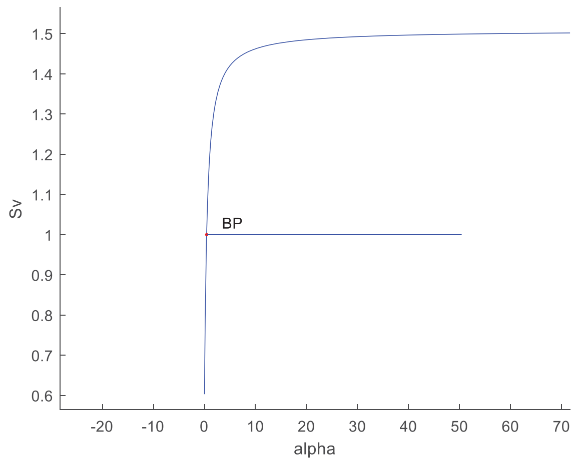

Figure 2a: (Bifurcation diagram for problem 2).

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This resulted in the Utopia point of 0 and the MNLMPC values of the control variables obtained were $[u_1, u_2, u_3]$ = [0.0004, 0.0405, 0.5362]. The various MNLMPC profiles are shown in Figures 2b-2h.

Two problems involving drug addiction models have been shown to exhibit branch points leading to two different solution branches. In both cases, it is computationally shown that the MNLMPC calculations would converge to the Utopia solution as the theoretical analysis predicts.

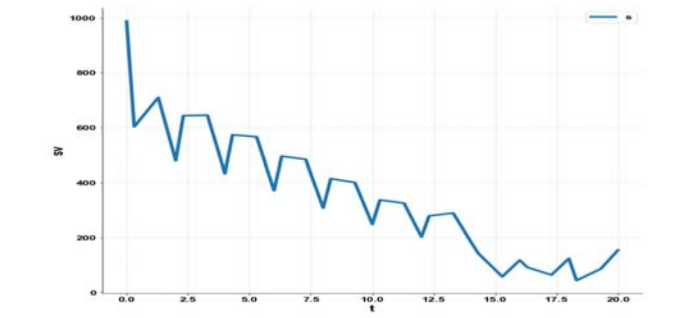

Figure 2b: MNLMPC diagram for problem 2 (sv vs. t).

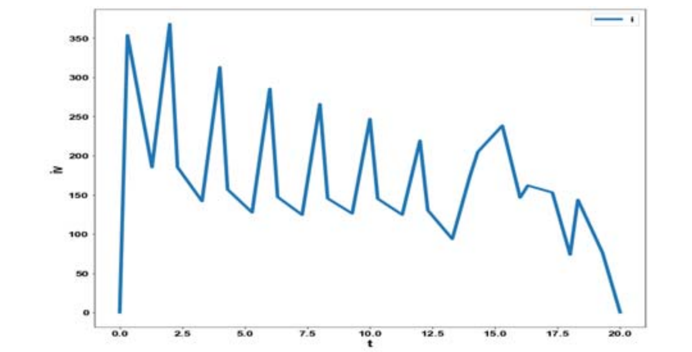

Figure 2c: MNLMPC diagram for problem 2 (iv vs. t).

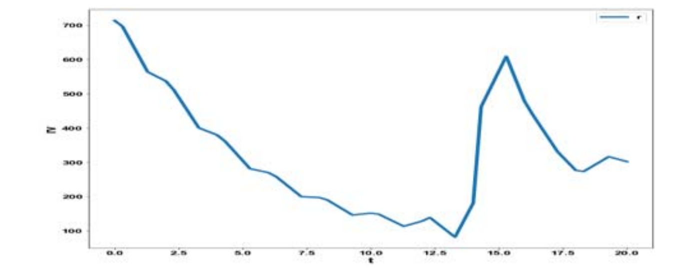

Figure 2d: MNLMPC diagram for problem 2 (rv vs. t).

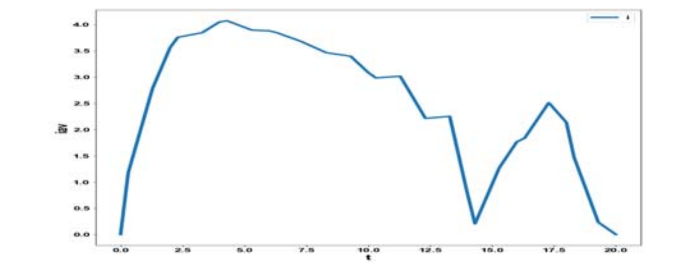

Figure 2e: MNLMPC diagram for problem 2 (mv vs. t).

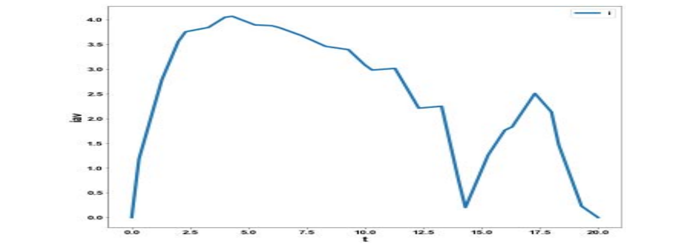

Figure 2f: MNLMPC diagram for problem 2 (iav vs.t).

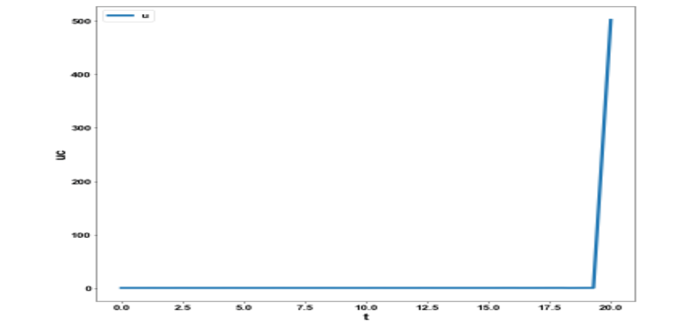

Figure 2g: MNLMPC diagram for problem 2 (uc vs. t).

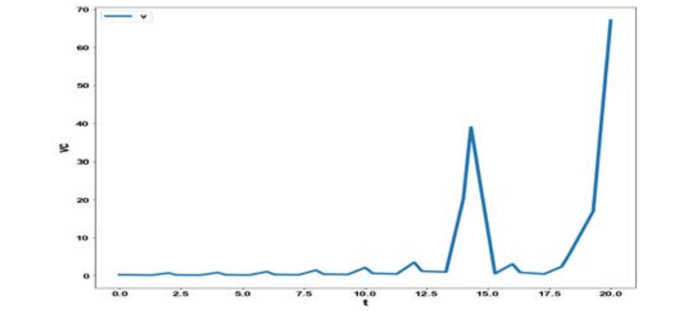

Figure 2h: MNLMPC diagram for problem 2 (vc vs. t).

Conclusions and future work

Branch points leading to two separate branches were exhibited when bifurcation analysis was performed on the two drug addiction models considered in this paper. The rigorous analysis demonstrated that the presence of the branch points would result in the MNLMPC calculations converging to the Utopia solution. This fact was also computationally validated. The presence of the branch points indicates that there are two paths that the drug abuse dynamics can take. The attainment of the Utopia point shows that the number of light and heavy drug users can both be minimized while maximizing the number of users who do not take drugs. Future work would involve using drug addiction models with time delay.

Data availability statement

All data used is presented in the paper.

References

1. Bae Y. Chaotic dynamics in tobacco's addiction model. Int J Fuzzy Logic Intell Syst. 2014;14(4):322-331.
Available from: https://doi.org/10.5391/IJFIS.2014.14.4.322

Citation: Sridhar LN. Bifurcation Analysis and Multiobjective Nonlinear Model Predictive Control of Drug Addiction Models. J Cardiovasc Med Cardiol. 2024;11(4):096-102. Available from: https://dx.doi.org/10.17352/2455-2976.000215

2. Mushayabasa S, Bhunu CP. Epidemiological consequences of non-compliance to HCV therapy among intravenous drug users. Int J Res Rev Appl Sci. 2011;8(3):288-295. Available from: https://www.researchgate.net/profile/Steady-Mushayabasa/publication/247967360_Epidemiological_consequences_of_non-compliance_to_HCV_therapy_among_intravenous_drug_users/links/00b7d5285d94cc6830000000/Epidemiological-consequences-of-non-compliance-to-HCV-therapy-among-intravenous-drug-users.pdf

3. Mushayabasa S. The role of optimal intervention strategies on controlling excessive alcohol drinking and its adverse health effects. J Appl Math. 2015;2015:238784. Available from: https://doi.org/10.1155/2015/238784

4. Mushayabasa S, Tapedzesa G. Modeling illicit drug use dynamics and its optimal control analysis. Comput Math Methods Med. 2015;2015:383154. Available from: https://doi.org/10.1155/2015/383154

5. Hasan M, Shahin ASM. Drug rehabilitation center based survey on drug dependence in Dhaka city. Update Dent Coll J. 2013;3(1):32-36. Available from: http://dx.doi.org/10.3329/updcj.v3i1.17982

6. Islam MA, Biswas MHA. Mathematical analysis of dynamic model of drug addiction in Bangladesh. Abstr Proc Int Conf Adv Comput Math. 2017. Department of Mathematics, University of Dhaka, Bangladesh. Available from:

7. Islam MA, Biswas MH. Optimal control strategy applied to a dynamic model of drug abuse incident for reducing its adverse effects. 2020;05. Available from: https://doi.org/10.1101/2020.05.02.20088468

8. Lavi O, Gottesman MM, Levy D. The dynamics of drug resistance: A mathematical perspective. Drug Resist Updat. 2012;15(1-2):90-97. Available from: https://doi.org/10.1016/j.drup.2012.01.003

9. Nyabadza F, Njagarah JBH, Smith RJ. Modelling the dynamics of crystal meth ('Tik') abuse in the presence of drug-supply chains in South Africa. Bull Math Biol. 2013;75(1):24-48. Available from: https://link.springer.com/article/10.1007/s11538-012-9790-5

10. White E, Comiskey C. Heroin epidemics, treatment and ODE modeling. Math Biosci. 2007;208(1):312-324. Available from: https://doi.org/10.1016/j.mbs.2006.10.008

11. Isa RS, Emmanuel S, Danat NT, Abubakar SS, Hwere TS, Garba U. Mathematical modeling of illicit drug use dynamics examining the impact of recycling recovered individuals into the population. Appl Math Comput Intell (AMCI). 2024;13(2):74-99. Available from: https://doi.org/10.58915/amci.v13i2.226

12. Donoghoe M. Illicit drugs. In: Murray CJL, Lopez AD, editors. Quantifying Global Health Risks: The Burden of Disease Attributable to Selected Risk Factors. Cambridge, Mass: Harvard University Press; 1996.

13. Murray RM, Morrison PD, Henquet C, Di Forti M. Cannabis, the mind, and society: the hash realities. Nat Rev Neurosci. 2007;8(11):885-895. Available from: http://dx.doi.org/10.1038/nrn2253

14. Pluddemann A, Dada S, Parry C. Monitoring alcohol and drug abuse trends in South Africa, South Africa Community Epidemiology Network on Drug Use (SACENDU). SACENDU Res Brief. 2008;11(2).

15. McGinty EE, Daumit GL. Integrating mental health and addiction treatment into general medical care: the role of policy. Psychiatr Serv. 2020;71:1163–1169. Available from: https://doi.org/10.1176/appi.ps.202000183

16. Hooker SA, Sherman MD, Lonergan-Cullum M, Sattler A, Liese BS, Justesen K, et al. Mental health and psychosocial needs of patients being treated for opioid use disorder in a primary care residency clinic. J Prim Care Community Health. 2020;11:2150132720932017. Available from: https://doi.org/10.1177/2150132720932017

17. Butler A, Nicholls T, Samji H, Fabian S, Lavergne MR. Prevalence of mental health needs, substance use, and co-occurring disorders among people admitted to prison. Psychiatr Serv. 2021;73:737–744. Available from: https://doi.org/10.1176/appi.ps.202000927

18. Scott CK, Dennis ML, Grella CE, Mischel AF, Carnevale J. The impact of the opioid crisis on U.S. state prison systems. Health Justice. 2021;9:17. Available from: https://doi.org/10.1186/s40352-021-00143-9

19. Chang JE, Franz B, Pagán JA, Lindenfeld Z, Cronin CE. Substance use disorder program availability in safety-net and non-safety-net hospitals in the US. JAMA Netw Open. 2023;6:e2331243. Available from: https://doi.org/10.1001/jamanetworkopen.2023.31243

20. Paquette CE, Daughters SB, Witkiewitz K. Expanding the continuum of substance use disorder treatment: nonabstinence approaches. Clin Psychol Rev. 2022;91:102110. Available from: https://doi.org/10.1016/j.cpr.2021.102110

21. Akanni JO, Olaniyi S, Akinpelu FO. Global asymptotic dynamics of a nonlinear illicit drug use system. J Appl Math Comput. 2021;66:39-60. Available from: https://doi.org/10.1007/s12190-020-01423-7

22. A, A., Akanni JO. Dynamics of illicit drug use and banditry population with optimal control strategies and cost-effectiveness analysis. Comput Appl Math. 2022;41:Paper No. 53, 37. Available from: https://doi.org/10.1007/s40314-022-01760-2

23. Olaniyi S, Akanni JO, Adepoju OA. Optimal control and cost-effectiveness analysis of an illicit drug use population dynamics. J Appl Nonlinear Dyn. 2023;12:133-146. Available from: https://www.lhscientificpublishing.com/Journals/articles/DOI-10.5890-JAND.2023.03.010.aspx

24. Dhooge A, Govaerts W, Kuznetsov AY. MATCONT: A Matlab package for numerical bifurcation analysis of ODEs. ACM Trans Math Softw. 2003;29(2):141-164. Available from: https://doi.org/10.1145/779359.779362

25. Dhooge A, Govaerts W, Kuznetsov YA, Mestrom W, Riet AM. CL_MATCONT; A continuation toolbox in Matlab. 2004. Available from: http://dx.doi.org/10.1145/952532.952567

26. Kuznetsov YA. Elements of applied bifurcation theory. New York: Springer; 1998. Available from: https://www.ma.imperial.ac.uk/~dturaev/kuznetsov.pdf

27. Kuznetsov YA. Five lectures on numerical bifurcation analysis. Utrecht; 2009.

28. Govaerts WJF. Numerical Methods for Bifurcations of Dynamical Equilibria. Philadelphia: SIAM; 2000. Available from: https://epubs.siam.org/doi/10.1137/1.9780898719543

29. Sridhar LN. Elimination of oscillations in fermentation processes. AIChE J. 2011;57(9):2397-2405. Available from: https://doi.org/10.1002/aic.12457

30. Flores-Tlacuahuac A, Morales P, Riveral Toledo M. Multiobjective nonlinear model predictive control of a class of chemical reactors. Ind Eng Chem Res. 2012;17:5891-5899. Available from: https://doi.org/10.1021/ie201742e

31. Sridhar LN. Multiobjective optimization and nonlinear model predictive control of the continuous fermentation process involving Saccharomyces cerevisiae. Biofuels. 2022;13:249-264. Available from: https://doi.org/10.1080/17597269.2019.1674000

32. Miettinen KM. Nonlinear Multiobjective Optimization. Kluwer International Series; 1999. Available from: https://books.google.co.in/books/about/Nonlinear_Multiobjective_Optimization.html?id=ha_zLdNtXSMC&redir_esc=y

33. Hart WE, Laird CD, Watson JP, Woodruff DL, Hackebeil GA, Nicholson BL, Siirola JD. Pyomo – Optimization Modeling in Python. 2nd ed. 67;2017. Available from: https://link.springer.com/book/10.1007/978-3-319-58821-6

34. Biegler LT. An overview of simultaneous strategies for dynamic optimization. Chem Eng Process Process Intensif. 2007;46:1043-1053. Available from: https://doi.org/10.1016/j.cep.2006.06.021

35. Wächter A, Biegler L. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. Math Program. 2006;106:25-57. Available from: https://doi.org/10.1007/s10107-004-0559-y

36. Tawarmalani M, Sahinidis NV. A polyhedral branch-and-cut approach to global optimization. Math Program. 2005;103(2):225-249. Available from: https://link.springer.com/article/10.1007/s10107-005-0581-8

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